Coordinate Geometry 2

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Other norms :- $x = (x_1, x_2, x_3, \dots, x_n) \in \mathbb{R}^n \Rightarrow x_i \in \mathbb{R}$

Types

- $||x|| = x_1 + x_2 + \dots + x_n$
- $||x|| = |x_1 + x_2 + \dots + x_n|$
- $||x||_1 = \sqrt[1]{|x_1^1 + x_2^1 + \dots + x_n^1|}$
- $||x||_2 = \sqrt[2]{|x_1^2 + x_2^2 + \dots + x_n^2|}$ → Euclidean norm
- $||x||_m = \sqrt[n]{|x_1^m + x_2^m + \dots + x_n^m|}$

$\sup(\text{set } A) = \text{suprimum}(\text{set } A) = x \Rightarrow$ if $y \in A$ then $y \leq x$

$\inf(\text{set } A) = \text{infimum}(\text{set } A) = x \Rightarrow$ if $y \in A$ then $y \geq x$

$$\text{sup-norm} = ||x||_\infty = \sqrt[\infty]{|x_1^\infty + x_2^\infty + \dots + x_n^\infty|} \Rightarrow = \boxed{|\max(x_i)|}$$

$$\sqrt[\infty]{1^\infty + 2^\infty} \Rightarrow 2$$

$$\sqrt[\infty]{5^\infty + 9^\infty} \Rightarrow 9$$

$$\sqrt[\infty]{1^\infty + 2^\infty + 3^\infty} \Rightarrow 3$$

$$\left(\frac{2}{3}\right)^\infty \longrightarrow 0$$

Definition (Norm) :- (simple definition for high school students)

using these ↪ $(0,0,1), (0,1,0)$
$(0, a, b), \; a, b \in \mathbb{R}$

Given a vector space X defined over $\mathbb{R}$ or $\mathbb{C}$ a norm of X is a real valued function $d : X \to \mathbb{R}$ with the following properties :-

(i) Triangle Inequality :-

$$d(x+y) \leq d(x) + d(y) \quad \forall x, y \in X$$

(ii) Positiveness :- $\forall x \in X, \; d(x) = 0 \Rightarrow x = 0$

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(iii) Absolute homogeneous :- $d(sx) = |s|\, d(x)\ \forall x \in X$ and s is a scalar

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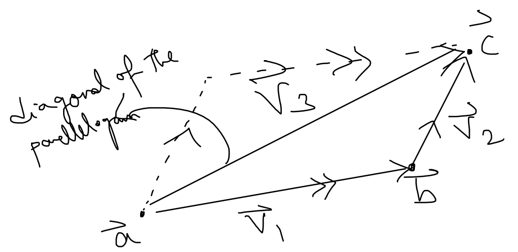
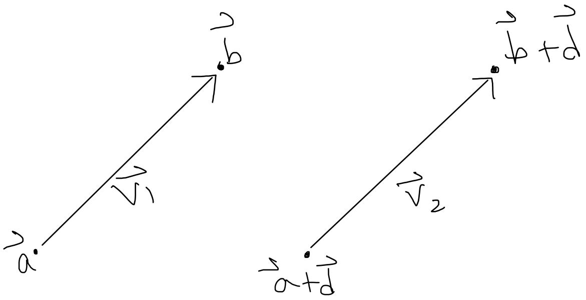


$\vec{v}_3 = \vec{v}_1 + \vec{v}_2$

$\|\vec{v}_3\| = \|\vec{v}_1 + \vec{v}_2\| \rightarrow$ depends on the angle between $\vec{v}_1$ & $\vec{v}_2$



$$\vec{v}_2 = \vec{b}+\vec{d} - (\vec{a}+\vec{d}) = \vec{b} - \vec{a} = \vec{v}_1$$

$$\implies \vec{v}_1 = \vec{v}_2$$

$\vec{a} = (a_1, a_2, \ldots, a_n)$ $\quad \vec{d} = (d_1, d_2, \ldots, d_n)$

$\vec{b} = (b_1, b_2, \ldots, b_n)$

$$\vec{a}+\vec{d} = (a_1+d_1, \ldots, a_n+d_n)$$

$$\vec{b}+\vec{d} = (b_1+d_1, \ldots, b_n+d_n)$$

$$(\vec{b}+\vec{d}) - (\vec{a}+\vec{d}) = (b_1+d_1-a_1-d_1, \ldots, b_n+d_n-a_n-d_n)$$

$$= (b_1-a_1, \ldots, b_n-a_n) = \vec{b} - \vec{a}$$

$\implies$ So, vectors are not location specific

Two vectors are equal iff their magnitude and direction are same

$$\vec{a} = \vec{b} \implies \vec{a} - \vec{0} = \vec{b} - \vec{0} \implies \|\vec{a}\| = \|\vec{b}\| \text{ and } \frac{\vec{a}}{\|\vec{a}\|} = \frac{\vec{b}}{\|\vec{b}\|}$$

Vectors that are defined from origin are called position vectors

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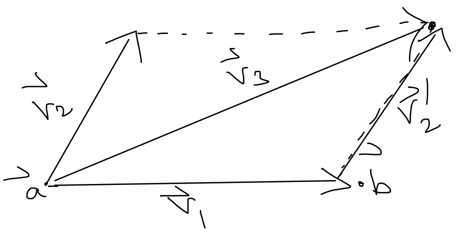
$\vec{v} = (v_1, v_2, \ldots, v_n) \xrightarrow[\text{to}]{\text{transformed}} \vec{w} = (w_1, w_2, \ldots, w_n)$

$T(\vec{v}) = \vec{w}$

$\rightarrow$ T is a function known as transformer.

Any linear change can be shown as multiplication by matrices

Linear change $\Rightarrow$ $w_i = \sum a_i v_i$ where $a_i \in \mathbb{R}$



$\vec{v_3} = \vec{v_1} + \vec{v_2}$

$\vec{v_2}' = \vec{v_2}$ as it is just parallel shift

$\Rightarrow \vec{v_3} = \vec{v_1} + \vec{v_2}'$ as well